cheaper construction of the post-coupled oscillator is often adequate.

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References

[1] B. C. Taylor, S. J. Fray, and S. E. Gibbs, "Frequency saturation effects in T.E.O.s," *IEEE Trans. Microwave Theory Tech.*, vol. 18, Nov. 1970.
[2] A. A. Sweet, "How to build a Gunn oscillator," *Microwave Ass. Micronotes*, vol. 11, Mar. 1974.
[3] R. L. Eisenhart and P. J. Kahn, "Theoretical and experimental analyses of a waveguide mounting structure," *IEEE Trans. Microwave Theory Tech.*, vol. 19, Aug. 1971.
[4] J. F. White, "Simplified theory for post coupling Gunn diodes to waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. 20, June 1972.
[5] F. L. Warner and G. S. Hobson, "Loaded Q-factor measurements on Gunn oscillators," *Microwave J.*, Feb. 1970.
[6] *Waveguide Handbook*, N. Marcuvitz, Ed. (M.I.T. Rad. Lab. Series), vol. 10, 1949.
[7] R. A. Gough, "Varactor tuned Gunn effect oscillators," Ph.D. thesis, Bradford University, 1973.
[8] R. Davies, "Basic design principles of frequency stable microwave oscillators," *Commun. Int.*, Jan. 1975.
[9] J. A. Copeland, "Theoretical study of a Gunn diode in a resonant circuit," *IEEE Trans. Electron Devices*, vol. 14, Feb. 1967.

Radiation Losses of Planar Circuit Resonators and the R/Q Parameter

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***Abstract*—The resonator parameter R/Q, the ratio of a shunt resistance to the unloaded Q, which might be termed a "mode-geometry" parameter, is a natural parameter for characterizing oscillation modes of planar-circuit resonators which are open circuited at the edges. These resonators are often excited by connection at the edge to a microstrip transmission line, and the appropriate shunt resistance is the equivalent resistance at resonance at these terminals for that mode.**

Losses in planar-circuit resonators include the ohmic (skin-effect and dielectric) losses of enclosed resonators plus a radiation-loss component. For a variety of planar resonators, the ohmic losses are easily calculated, but the radiation-loss determination is a difficult boundary-value problem. More specifically, the determination of either the unloaded Q or the radiation component of the unloaded Q is often readily accessible only through measurement. The knowledge of the R/Q parameter allows one, in effect, to replace the Q measurement with a shunt-resistance measurement, which is often more expedient to perform.

Two simple planar-resonator configurations, the circular disk, and the square plate are studied. The radiation component of the Q is evaluated by using the measured shunt resistance and the R/Q parameter to calculate the total unloaded Q and thence the radiation component of the unloaded Q. A comparison is made between these results and those obtained from direct Q measurements.

Introduction

The direct calculation of radiation loss from planar circuits is an extremely difficult boundary-value problem. Radiation from microstrip can be treated by regarding the strip as a line source of current [1] and an extension of this approach has been used [2]; however, even the simplest planar resonator, the circular disk, is not amenable to this approach. Alternative routes toward a solution of the problem of losses in open resonators will be useful.

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The Geometrical Cavity Factor R/Q

Resonators having a Q, which is sufficiently large so that the configuration of the electromagnetic field is substantially the same as in the absence of losses, can be characterized by a ratio of a shunt resistance to a Q which is essentially geometrical, i.e., it is independent of cavity losses [3]. In the case of an open-circuited planar resonator it is convenient to define the shunt resistance in terms of the voltage at the point at which it will be excited by connection to a microstrip line, typically at the edge of the resonator. Thus, if the total time-average power dissipation due to conductor losses, dielectric losses, and radiation losses is P and the peak amplitude of the voltage at the excitation point is V, the shunt resistance is

$$R = \frac{V^2}{2P}. \tag{1}$$

The unloaded Q is

$$Q = \omega_0 \frac{U}{P} \tag{2}$$

where U is the energy stored in the cavity and P is the previously mentioned power dissipation at resonance; whence

$$R/Q = \frac{V^2}{2\omega_0 U}. \tag{3}$$

This is a result which is dependent on the mode type but independent of losses as long as they do not cause significant modifications to the spatial distribution of the fields within the cavity. It is valid for any planar configuration, whether it is open circuited or short circuited at the edges of the plane, to the extent that edge effects represent a small perturbation to the otherwise known field distribution.

For a cavity for which the shunt resistance can be measured and the right-hand side of (3) can be calculated, the Q can then be found and, if the Q for conductor loss and dielectric loss can be determined by other means, the radiation loss can be found from the fact that

$$\frac{1}{Q} = \frac{1}{Q_{\text{cond. loss}}} + \frac{1}{Q_{\text{diel. loss}}} + \frac{1}{Q_{\text{rad. loss}}}. \tag{4}$$

Examples

The preceding approach can be readily used with open-circuited planar resonators of simple geometrical configuration.

1) Circular Disk: One such resonator is the circular disk which is illustrated in a cross-sectional view in Fig. 1. In the dominant mode, it contains an electric field lying in the z direction only and given by

$$E_z = E_0 \cos \phi J_1(kr) \tag{5}$$

where r and ϕ are the usual circular cylindrical coordinates. The open-circuit edge condition requires that $J_1'(ka) = 0$ or $ka = 1.841$, hence the edge voltage $V = E_0 b J_1(1.841)$. There are two components of the magnetic field, H_ϕ and H_r, which are not needed for (3) but are needed to separate the losses into their components. The energy stored in this mode is obtained by integrating the energy density $\frac{1}{2}\varepsilon|E|^2$ over the cavity volume and gives the result

$$U = E_0^2 \frac{\pi}{4} \varepsilon b a^2 \left(1 - \frac{1}{(ka)^2}\right) J_1^2(ka) \tag{6}$$

whence

$$\frac{R}{Q} = \frac{2}{\pi} \eta \frac{b}{a} \frac{ka}{(ka)^2 - 1} \tag{7}$$

TABLE I

Planar Circuits, ε_r = 9.7, b = 0.025 inches	Theo. f_{res}	Meas. f_{res}	Theo. (R/Q)	Meas. R	Calc. Q_R	Meas. Q_m	Cal. Q_{rad_R}	Calc. Q_{rad_m}
Circular Disc a = 0.75 inches								
Dominant Mode	1.48	1.466	1.98	301	152	179	304	440
Lowest n=0 Mode	3.08	3.005	0.334	90.4	270	280	750	840
Second Mode	2.46	2.455	1.471	363	247	320	705	2000
Square Plate 2a = 1.28 inches								
Dominant Mode	1.48	1.461	3.01	491	163	177	352	425
Second Mode	2.10	2.100	2.13	549	258	314	945	2800

Each tabulated value of Q_m is the average of 4 values representing amplitude data and phase data for two different resonators. Q_{rad_R} is the radiation component of Q calculated from Q_R and Q_{rad_m} is that calculated from Q_m.

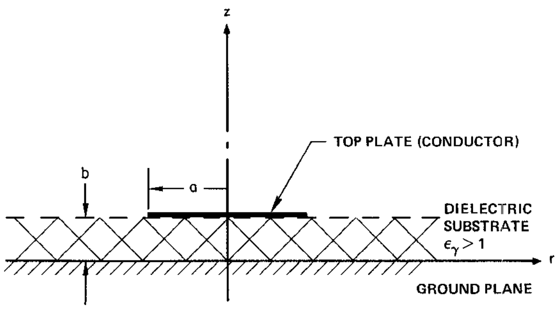

Fig. 1. Cross section of planar disk resonator.

where η is the intrinsic impedance of the dielectric substrate material, $377/\sqrt{\varepsilon_r}$.

Two planar disks of radius 0.75 in were fabricated on an alumina (ε_r = 9.7) substrate of thickness 0.025 in, which gives, from (7), R/Q = 1.98 Ω at a calculated resonant frequency of 1.483 GHz. The disks were excited from a 50-Ω microstrip line which was in turn connected to a 50-Ω coaxial line. The shunt resistance at resonance was measured using an H-P network analyzer, with the data transformed to the reference plane at the edge of the disk. The measured shunt resistance was 311 Ω for one and 290 for the second for an average value of 301 Ω at the measured resonant frequency of 1.466 GHz. Using the calculated R/Q value, this yields a Q of 301/1.98 = 152.

The conducting surfaces consist of a 200-Å layer of nickel (μ_R = 110, $\sigma = 1.42 \times 10^7$ mhos/m) backed with a relatively thick layer of gold ($\sigma = 4.54 \times 10^7$ mhos/m). The surface resistance of this composite is readily computed [4] and results in a Q due to conductor losses of 323. Combining this with a dielectric loss tangent for alumina of 0.0002 allows these two ohmic losses to be removed from the total loss to give, using (4)

$$\frac{1}{Q_{rad}} = \frac{1}{152} - \left(\frac{1}{323} + 0.0002\right)$$

or

$$Q_{rad} = 304.$$

The preceding determination of the unloaded Q and the radiation component Q_{rad} was compared with direct measurements of the Q using first the ratio of the resonant frequency to the frequency band between points at which the magnitude of the impedance at the edge of the disk was 70.7 percent of the maximum impedance, and secondly the ratio of the resonant frequency to the frequency band between points at which the phase angle of the input impedance was ±45°. The results based on impedance magnitude data were 1466/8.4 = 174 and 1466/7.9 = 185 for the first and second disk, respectively; from the phase-angle data the results were 1466/8.6 = 171 and 1466/7.9 = 185, respectively. The average of these results gives an unloaded Q of 179, from which the removal of the two ohmic loss components as before leaves a remaining Q_{rad} of 440. Thus, to recapitulate, the shunt resistance determined Q of 152 is to be compared with a directly measured Q of 179; the first leads to a Q_{rad} of 304 and the second to a Q_{rad} of 440.

For higher circular-disk modes for which

$$E_z = E_0 \cos n\phi J_n(kr) \quad J_n'(ka) = 0$$

the result which is the generalization of (7) is

$$\frac{R}{Q} = \frac{2}{\pi}\frac{1}{1+\delta_0{}^n}\eta\frac{b}{a}\frac{ka}{(ka)^2 - n^2}, \qquad n = 0, 1, 2 \cdots \tag{8}$$

where $\delta_0{}^n$ is unity when $n = 0$ and is zero otherwise.

This result has again been checked for the lowest $n = 0$ mode for the two planar disks whose dimensions have been given. In this case, the evaluation of (8) gives R/Q = 0.334 which, combined with the average measured shunt resistance of 90.4 Ω, gives a Q of 270 at a measured frequency of 3.005 GHz. Using (4) in the same manner as before then yields a result for Q_{rad} of 750.

A direct measurement of the Q of the lowest $n = 0$ mode from the 70.7-percent response points and from the ±45° phase-shift point produced an average result of Q = 280, which yields a radiation component Q_{rad} = 840.

A third mode, the lowest $n = 2$ mode, was examined in the same way and the results are shown in Table I. These results are less encouraging, presumably because of the higher standing-wave ratio combined with the predominance of the ohmic losses. A high VSWR makes it more difficult to achieve high accuracy in spite of careful calibration of the network analyzer. The predominance of ohmic losses makes the radiation-loss calculation even more sensitive to errors in the total loss, since the radiation loss is the difference between the total and the ohmic terms.

2) *Square Plate:* A second configuration for which the R/Q parameter may readily be found is the square plate excited at one corner. Fig. 1 may again be used to illustrate the resonator

in cross section, where $2a$ is now the dimension of either side of the square.

In the dominant mode the electric field with corner excitation is

$$E_z = E_0 \left[\sin \frac{\pi}{4a} (x + y) \cos \frac{\pi}{4a} (x - y) \right] \tag{9}$$

for which the resonance is at $ka = \pi/2$ and the mode-geometry parameter

$$R/Q = \eta \frac{b}{a} \frac{1}{ka}. \tag{10}$$

Squares of side $2a = 1.28$ in, which resonate at the same frequency as the circular disk, were fabricated. Using the same procedure as before, the average Q was found to be 163 compared with an average directly measured value of 177. Based on the value of 163, the Q due to radiation loss was found to be 352; based on the value of 177, the Q due to radiation loss was 425.

The electric field of the second square-plate mode is

$$E_z = E_0 \sin \frac{\pi}{2} \frac{x}{a} \sin \frac{\pi}{2} \frac{y}{a} \tag{11}$$

and resonates at

$$ka = \frac{\pi}{\sqrt{2}}.$$

The R/Q parameter in this case is again

$$\frac{R}{Q} = \eta \frac{b}{a} \frac{1}{ka} \tag{12}$$

which allows the comparisons made before. These results as well as the preceding square-plate results are similarily tabulated in the second portion of Table I. The previous comments regarding accuracy apply here also.

In the preceding examples, the Q's are high and the resonant frequencies widely separated so that there is considerable assurance that only the desired mode was excited. The correspondence between the measured and the calculated resonant frequencies should be noted.

Prior to performing the experimental work there was some concern regarding the possible influence of reflections from surrounding objects. Fortunately, these effects were found to be negligible as long as any perturbing objects were an inch or more from the edge of the resonator. This insensitivity is probably due to the relatively small thickness of substrate-to-wavelength ratio.

Summary

An additional route to the evaluation of radiation from planar-circuit cavity resonators is gained from a calculation of the R/Q parameter which, in effect, allows a Q measurement to be replaced by a resistance measurement. Preliminary results for two simple planar resonators are given.

References

[1] L. Lewin, "Radiation from discontinuities in stripline," *Proc. IEE*, vol. 107C, pp. 163–170, 1960.
[2] J. Watkins, "Radiation loss from open-circuited dielectric resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 636–639, Oct. 1973.
[3] E. L. Ginzton, *Microwave Measurements*. New York: McGraw-Hill, 1957, ch. 10.
[4] S. Ramo, J. R. Whinnery, and T. Van Duzer, *Fields and Waves in Communication Electronics*. New York: Wiley, 1965, pp. 298–300.

Measurement of Microwave Loss Tangent by Means of Microwave Resonator Bridge

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***Abstract*—A new cavity method of measurement of microwave loss tangent of a low-loss material is described. The method consists of the comparison of the reflection coefficient of the resonator filled with dielectric with the reflection coefficient of the reference resonator by means of microwave resonator bridge. Basic theoretical relations of a simplified resonator bridge are derived, a fundamental scheme of the measuring set is given, and some results of verified measurements presented. The obtained results confirm the suitability of the proposed method, especially for measuring low-loss dielectric materials, since the principle of comparison renders the method highly sensitive to small parameter differences of the resonator brought about by the measured sample.**

Nomenclature List

a_1	Equivalent voltage of the wave incident in port 1.
b_i	Equivalent voltage of the wave scattered from port i.
C	Capacitor of the equivalent series resonant circuit.
i	Integer denoting port of hybrid junction.
j	Square root of -1.
L	Inductor of the equivalent series resonant circuit.
P_1	Microwave power fed into the MRB in the unperturbed condition.
P_1'	Microwave power fed into the MRB in the perturbed condition.
P_4	Output power of the MRB in the unperturbed condition.
P_4'	Output power of the MRB in the perturbed condition.
R	Factor Z_{03}/Q_{03}.
Z_{02}, Z_{03}	Normalized geometrical factor of the measured or reference resonator, respectively.
Z_c	Characteristic impedance of the line.
Γ_2, Γ_3	Reflection coefficient of the measured or reference resonator, respectively.
Γ_4	Reflection coefficient of the detector.
Γ_{in}	Reflection coefficient of the MRB in the unperturbed condition.
Γ_{in}'	Reflection coefficient of the MRB in the perturbed condition.
δ	Angle of dielectric losses.
ε	Relative permitivity.
Θ	Phase shift.
θ	Relative difference of the resonant frequencies.
κ	Factor Q_{03}/Q_{02}.
ν	Relative detuning of the reference resonator.
χ	Factor defined by the expression (A9).
ω_{02}, ω_{03}	Resonance frequency of the measured or reference resonator, respectively.

I. Introduction

The cavity methods, especially the cavity perturbation methods, are often used to measure microwave dielectric losses, [1], [2], but in measuring materials of small dielectric losses it is very difficult to determine the change of the cavity quality factor by only the difference of the half-power bandwidth of the resonance curve, and the obtained results are thus unsatisfactory. The authors present a new and highly sensitive method for measuring

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